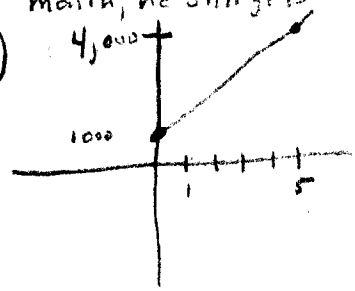
Math 131
October 11, 2000
Exam 1

Name Answer Key
Sec.
Instructor

1. A mainframe computer salesman's monthly salary (S) in dollars is a function of the number (n) of mainframes he sells that month. $S = f(n)$.
   a) Suppose that $f(0) = 1000$ and $f(5) = 4000$. Interpret this in plain English.
   b) Graph f assuming it is linear. What is its slope? Interpret the significance of the slope in plane English.

a) $f(0) = 1000$ means that even if the salesman sells no mainframes during the month, he still gets \$1000 - his base salary. If he sells 5 mainframes, he gets \$4000.

b) 

$\text{slope} = \frac{4{,}000 - 1000}{5 - 0} = \frac{3{,}000}{5} = \$600$ per mainframe.

The salesman gets \$600 commission for each mainframe he sells.

His salary for the month is \$1000 plus \$600 for each sale.

2. The population of India in 1948, just after independence was 350 million. At the end of the year 2000, India's population stood at 1 billion (1,000 million).
   a) What was the average rate of change of India's population from 1948 to 2000?
   b) Assuming linear growth at the rate in part a), calculate the population of India be at the end of 2008 (60 years after independence).

a) $\frac{1000 - 350}{2000 - 1948} = \frac{650 \text{ million}}{52 \text{ yrs}} = 12.5$ million per yr.

b) The population will grow 12.5 million each yr.

Population in 2008 = Population in 2000 + 8(12.5) = 1,000 + 100 millio
= 1.1 billion

3. Maggie's doctor prescribed three different drugs (A, B, and C) for her to take. Drug A decays exponentially in the body, B decays linearly, and C was neither linear nor exponential. Her blood was tested each hour to find out how much of each drug was present.

x (hours)	0	1	2	3	4
f(x) (mg)	18	21	20	17	12
g(x) (mg)	81	54	36	24	16
h(x) (mg)	37	34	31	28	25

a) Which one of these three functions represents drug A (exponential decay)? Which drug B (linear)? Briefly explain your answers.
b) Find possible equations for each of them.

a) $f(x)$ increases, then decreases, so it cannot represent either exp or linear decay

b) $h(x)$ decreases 3 mg per hour, so it represents linear decay (drug B.) (with slope -3)

$h(x) = 37 - 3t$

$g(x)$ represents exponential decay because the ratios of successive values of g are constant:

$\frac{54}{81} = .67$, $\frac{36}{54} = .67$, $\frac{24}{36} = .67$, $\frac{16}{24} = .67$.

$g(x) = 81(1-.33)^t = 81e^{-.40t}$

$.67 = (1-.33)$

If $.67 = e^k$, then

$\ln(.67) = k$

$-.40 = k$

4. Ivan will begin college in the fall of 2004. He wants to earn enough money by the fall of 2001 so that he can pay $10,000 of his freshman and sophomore tuition. He will set aside the money in a bank account that pays 8% annual interest compounded at the end of each year on September 1. What will he have to deposit on September 1, 2001, in order to have $10,000 on September1, 2004 and $10,000 on September, 2005?

8% annual interest

The present value of \$10,000 on Sept. 1, 2005 → Sept 1, 2001

$PV = \frac{10,000}{(1.08)^4} = \$7350.30$

PV of \$10,000 on Sept 1, 2004 → Sept 1, 2001

$\frac{10,000}{(1.08)^3} = \$7938.32$

Total to deposit on Sept 1, 2001 is

$7350.30 + 7938.32 = \$15,288.62$

5. Let $g(v)$ be the fuel efficiency of a car moving at v miles per hour, with efficiency measured in miles per gallon.
   a) Give the meaning, in plain English, of the equation $g(55) = 27$.
   b) Give the meaning, in plain English, of the equation $g'(55) = -0.54$ (include units). Why would g'(55) be negative?
   c) Using a) and b), approximate $g(58)$.

a) at 55 mph, the car's efficiency is 27 mpg.

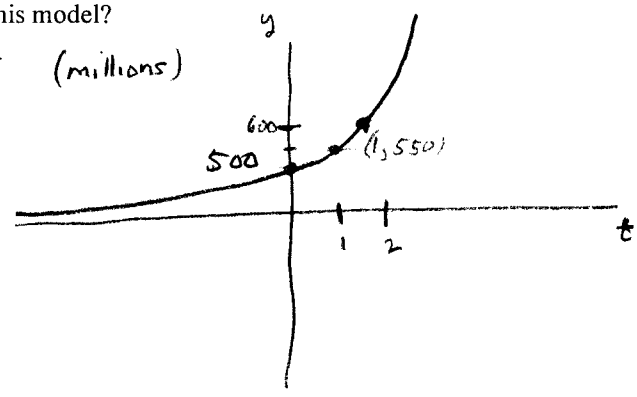
b) at 55 mph, the car loses efficiency at the rate of .54 mpg for each increase of 1 mph in the speed. g' is negative because a car going at a high rate of speed is usually less efficient than a car going at a more moderate pace.

c) $g(58) \approx g(55) + g'(55)(3) = 27 + 3(0.54) = 25.38$ mpg.

6. The number of bacteria in milk grows at a rate of 10% per day once the milk has been bottled. Typically, freshly bottled milk has a bacteria count of 500 million per bottle.
   a) Write an equation for $f(t)$, the number of bacteria t days after the milk is bottled.
   b) Graph $f(t)$, labeling the axes and intercepts.
   c) Suppose milk cannot be safely consumed if the bacteria count is greater than 3 billion (3,000 million) per bottle. How many days will the milk be safe to drink after it has been bottled, according to this model?

a) $y = f(t) = 500(1.10)^t$ (millions)

(b)



c) $3000 = 500(1.1)^t$

$\frac{3000}{500} = 1.1^t$

$6 = 1.1^t$

$\ln 6 = t \ln 1.1$

$\frac{\ln 6}{\ln 1.1} = t$

$t = 18.8$ days

7. A radioactive substance has a half-life of 5 years. If 2kg are present initially, how much will remain at the end of 10 years?

$k = \frac{\ln 2}{\text{half-life}} = .1386$

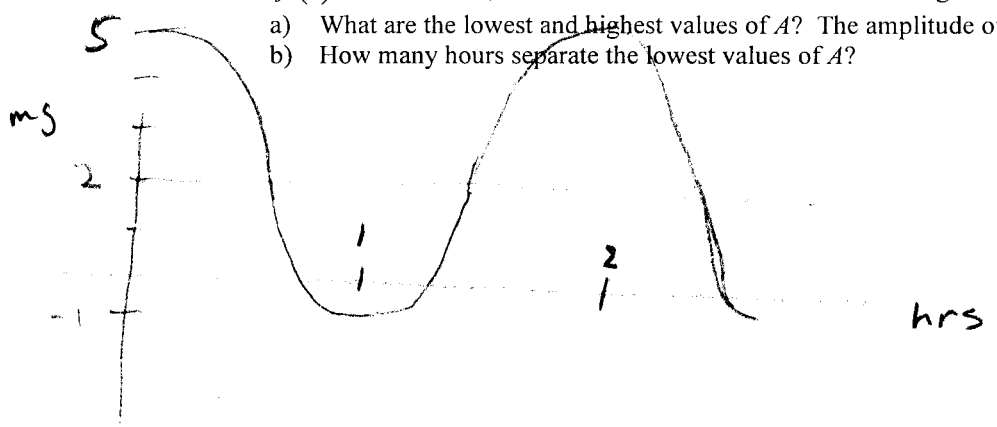
$A = 2e^{-.1386t}$ (kg)

$A = 2e^{-.1386 \cdot (10)} = 2e^{-1.386}$

$= .5$

Alternatively, After 5 yrs, the 2 kg decays to 1 kg. After another 5 yrs, the 1 kg becomes .5 kg.

8. Suppose that a chemical in the bloodstream is regulated so that the amount A is given by $A = f(t) = 2 + 3\cos \pi t$, where t is measured in hours and A in mg.
   a) What are the lowest and highest values of A? The amplitude of A?
   b) How many hours separate the lowest values of A?

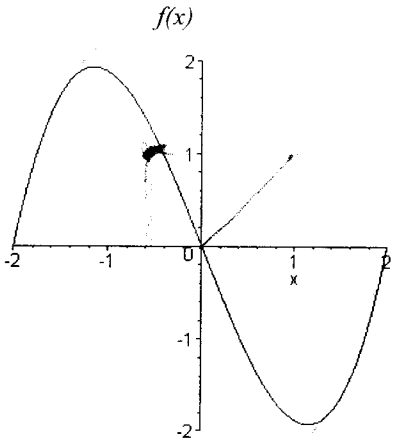


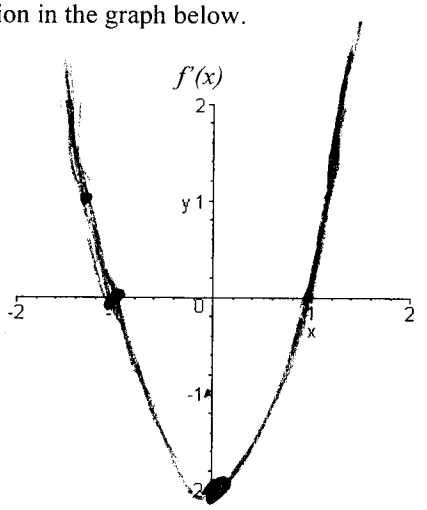
a) The graph ranges from 5 to -1. The amplitude is 3.

b) The period is $\frac{2\pi}{\pi} = 2$ hrs.

(The model should not have included negative values for the # of mg.s present in the bloodstream. That was our mistake.)

9. Sketch the graph of the derivative of the function in the graph below.





10. If $f(x) = x^3 + 4x$, estimate the value of $f'(3)$ to the nearest tenth by finding average rates of change over small intervals.

$\frac{f(3.01) - f(3)}{.01} = 31.09$

$\frac{f(3) - f(2.999)}{.001} = 30.99$

$\frac{f(3.0001) - f(3)}{.0001} = 31.00$

31.0